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Diversification Benefit Working Party

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The opinions expressed in this presentation and in the following slides are solely the views of the working party and do not represent that of the Singapore Actuarial Society or the respective companies.

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Special thanks to Aon Benfield for the generous sponsorship of Remetrica which was used extensively for this research.

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Objectives

"MAS looked into the possibility of recognising diversification benefits when aggregating the risk requirements under RBC 2. However, dependencies between different risks will vary as market conditions change and correlation has been shown to increase significantly during periods of stress or when extreme events occur. In the absence of any conclusive studies to show otherwise, MAS proposes not to take into account diversification effects for the aggregation of risk requirements under RBC 2."

Monetary Authority of Singapore, RBC 2 Consultation Paper June 2012

Objectives of Working Party:

- Study of correlations and dependency structures between various risk types
  - Risk types analysed: Insurance Risk, Investment Risk, Investment Credit Risk
  - Risk types not included in analysis: Operational Risk, Catastrophe Risk, Non-Investment Credit Risk, Group Risk etc
- Determine if diversification benefit exists even during stressed periods and estimate the amount of diversification benefit for a common insurer

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Agenda

Section 1.1: Concept and Methodology

- Correlation Coefficient, Dependency Structure and Risk Measure
- Impact of varying distributions
- Impact of varying portfolio mix and number of risk factors

Section 1.2: Practical Considerations

- Correlation Linear and Non-Linear dependency Structure
- Parametric Filtering Method
- Back-Testing Results

Section 2: Historical Data Analysis (Inter and Intra risk)

- Macroeconomic and Insurance factors
- Insurance Intra-Risk correlation
- Insurance and Investment Inter-Risk correlations
- Investment Intra-Risk correlation
- Confidence Interval of correlations

Section 3.1: Stochastic Impact Analysis – General Insurance

- Set up Pseudo Insurer
- Estimate diversification benefits
- Sensitivity Analysis

Section 3.2: Stochastic Impact Analysis – Life and Investments

- Diversification benefit among different portfolio mix
  - Regional Public Equities
  - Government Bonds vs Corporate Bonds
  - Corporate Bonds
  - Risky Assets vs. Fixed Income
  - Overall Portfolio Risk

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Section 1.1: Theory and Fundamentals

a. Understanding Correlation Coefficients

b. Understanding Dependency Structures

c. Understanding Risk Measures

d. Evaluating Dependency Structure on Risk Measure

e. Evaluating Loss Distribution on Risk Measure

f. Evaluating Multiple Risk Factors and Portfolio Mix on Risk Measure

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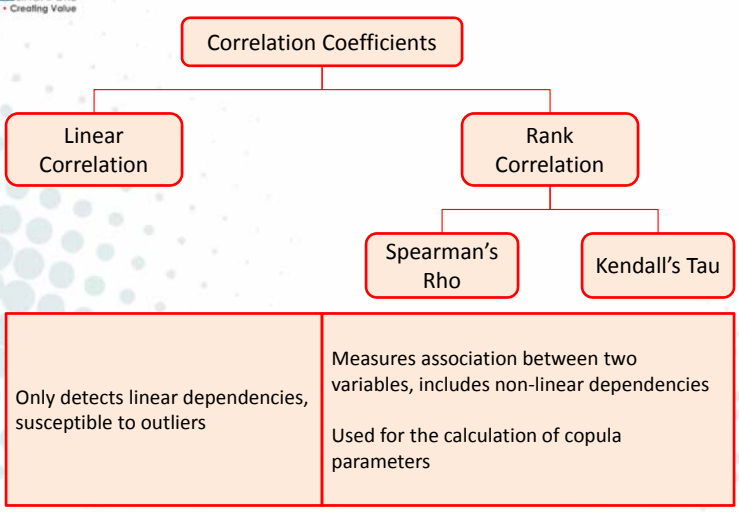
Section 1.1a: Correlation Coefficients



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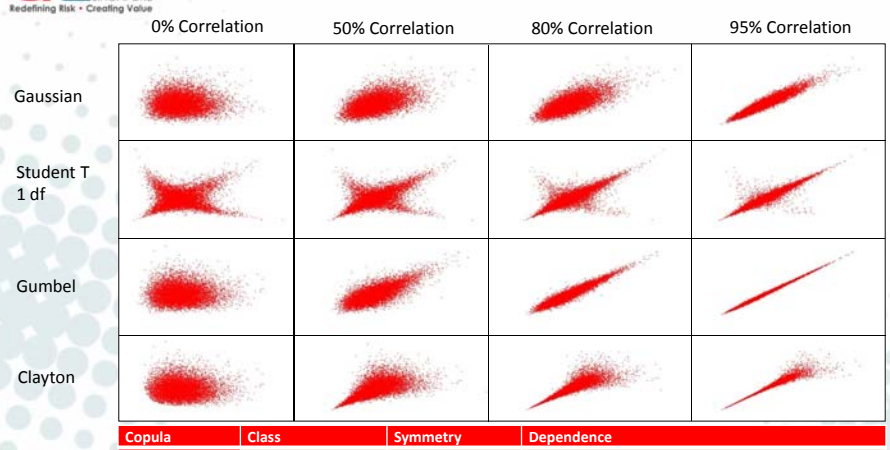
Section 1.1b: Dependency Structures



Copula	Class	Symmetry	Dependence
Clayton	Archimedean	Asymmetric	Strong Left Tail
Gumbel	Archimedean	Asymmetric	Strong Right Tail
Frank	Archimedean	Symmetric	Weak Tails, Strong Centre
FGM	Elliptical	Symmetric	Generally weak dependence
Gaussian	Elliptical	Symmetric	Moderate tail dependence
Student-t	Elliptical	Symmetric	Similar shape as Gaussian, stronger tail dependence

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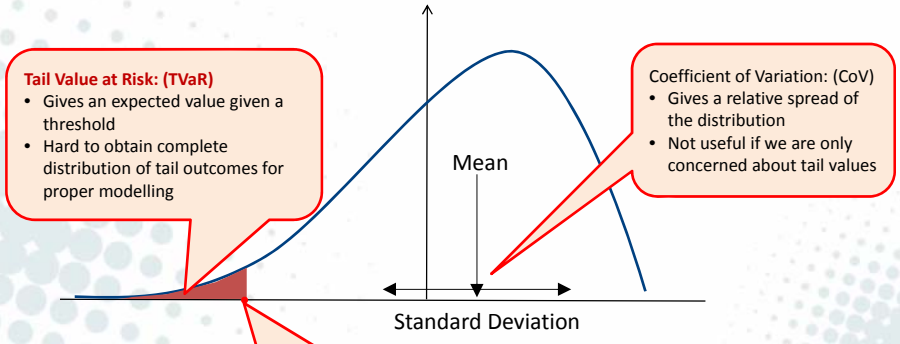
Section 1.1c: Risk Measures



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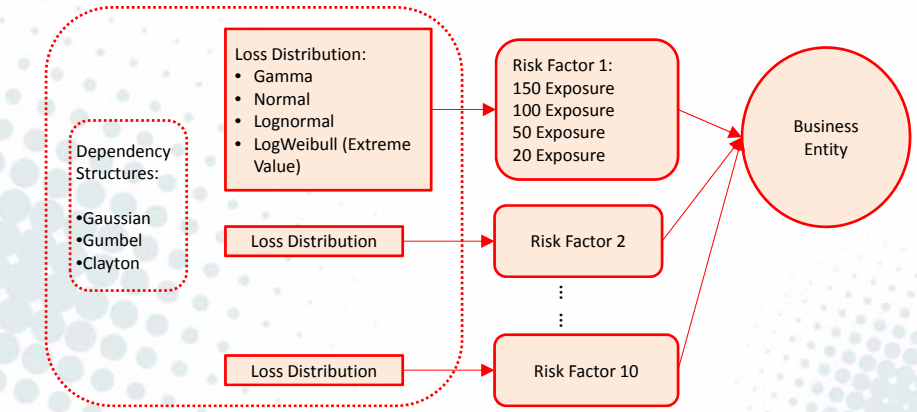
Section 1.1d: Model Used for Evaluation



Calculation of Diversification Benefit:

$$DB\% = 1 - \frac{Aggregate\ Losses\ at\ Entity\ Level}{RF\ 1\ Loss + RF\ 2\ Loss + \cdots + RF\ 10\ Loss}$$

Tested various copulas, distributions and varied the portfolio mixture as well as the number of risk factors so as to understand their impact on diversification benefit

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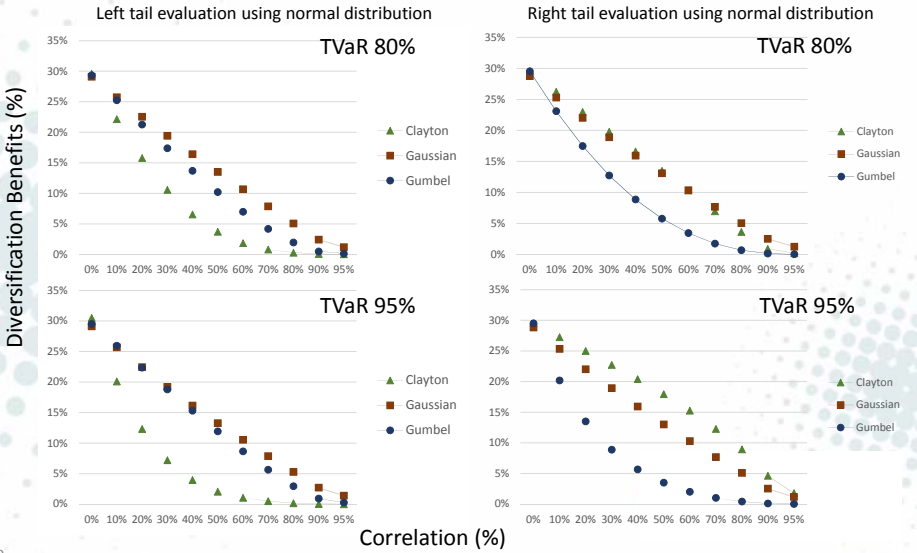
Section 1.1d: Evaluate Dependency Structure



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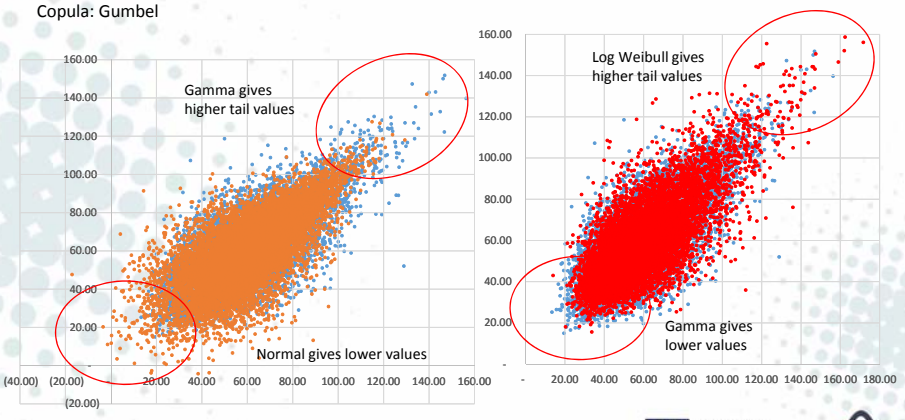
Section 1.1e: Evaluate Loss Distribution

Blue Dots: Gamma
Orange Dots: Normal
Red Dots: Log Weibull
Copula: Gumbel



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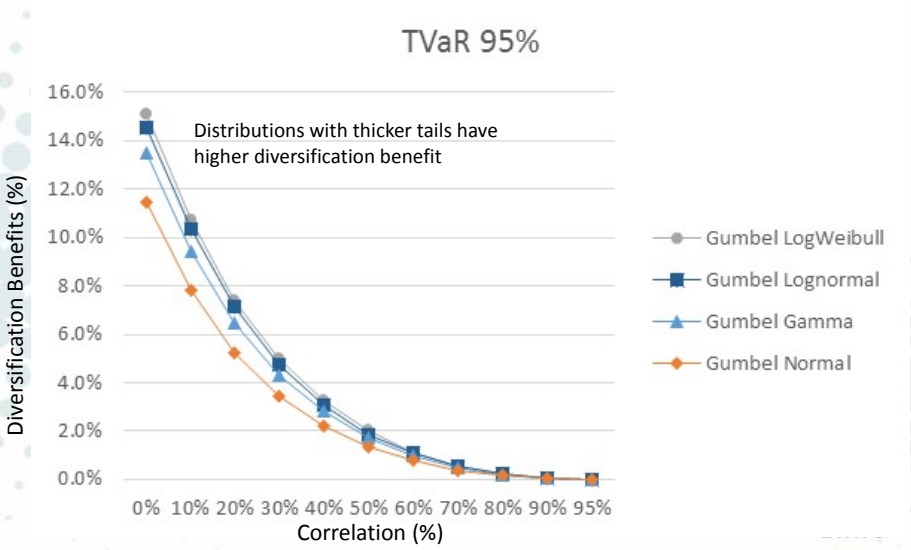
Section 1.1e: Evaluate Loss Distribution



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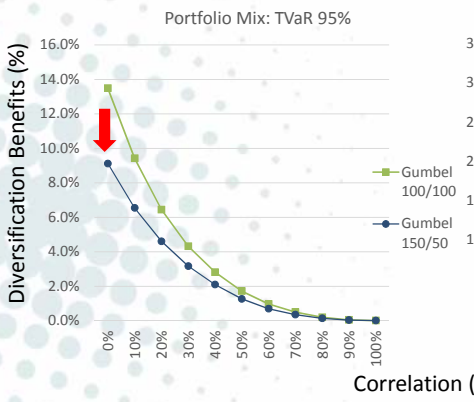
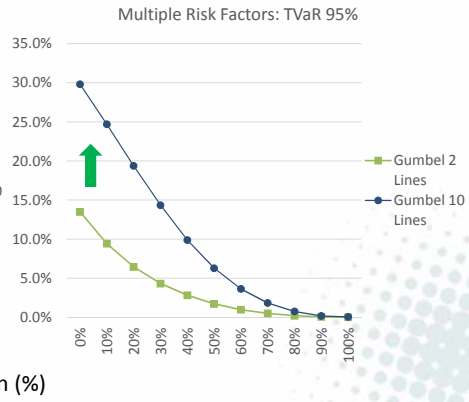
Section 1.1f: Portfolio Mix and Multi Risks



Uneven portfolio mix decreases the amount of diversification benefit



Increasing the number of risk factors increases the amount of diversification benefit

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Section 1.1: Conclusion

- **Dependency Structure**
  - Use Gumbel Copula when we model right tail dependency for loss distributions
  - Use Clayton Copula when we model left tail dependency for returns distributions
- **Loss Distribution**
  - Diversification benefit curve is largely determined by the dependency structure
  - Choice of underlying distribution has impact on the loss distribution, but not as significant compared to the choice of dependency structure
- **Uneven Portfolio Mix**
  - Presence of a dominant portfolio will reduce diversification benefit
  - For instance, changing portfolio mix from 50:50 to 60:40 will reduce the diversification benefit significantly
- **Multiple Risk Factors**
  - Increasing the number of risk factors will increase (explicit) diversification benefit
  - Segmenting the portfolio further gives more explicit diversification rather than implicit of a more grouped portfolio

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Section 1.2: Practical Considerations

Questions:

a. Is the relationship (e.g. within investment space) always linear?

b. How to ascertain a linear vs. non-linear relationship?

c. Is there any systematic way to filter the dependency structures to select the one that reflects the underlying data?

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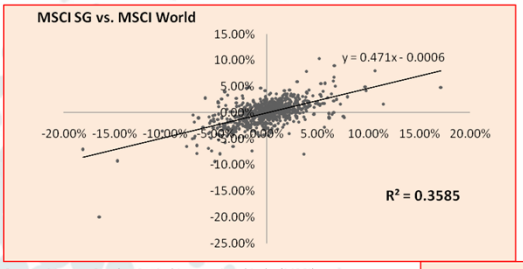
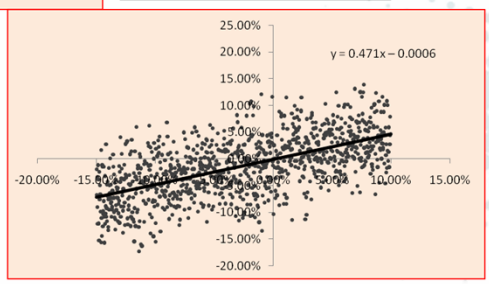
Section 1.2a: Linearity vs. Non-Linearity

Historical Log Return (1999-2013)



Source: Morgan Stanley Capital International Index(MSCI)

Corresponding Linear Form



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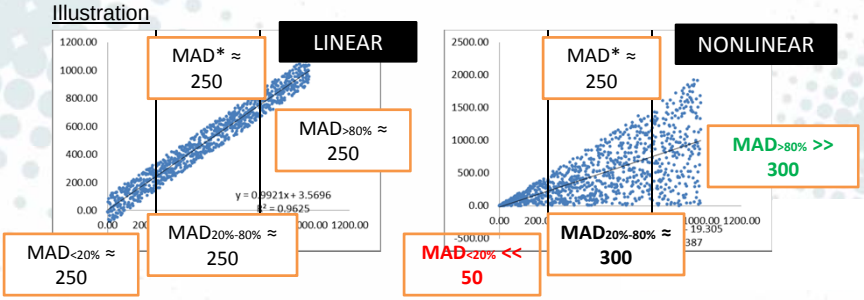
Section 1.2b: MAD Linearity Test

- Calculate **Mean Absolute Deviation** ("MAD")
  1. Regress variables Y against X using OLS: Y = mX + c
  2. Fit observed (data) X into equation to find expected y, denote as $\hat{Y}$
  3. Find residuals **$E(|Y-\hat{Y}|)$** = MAD* of the whole distribution
  4. Filter out selected top and bottom k percentile values for X (and middle ranges as well) e.g. <5,10,20 and >80,90,95 percentiles, 20-80 for middle
  5. Find each associated Y and $\hat{Y}$ and repeat steps above to find $MAD_{B/Tk\%}$ for these ranges
  6. Compare $MAD_{k\%}$ to MAD*. Set a threshold (e.g. 20% of MAD*) for the absolute difference between each MAD and $MAD_{k\%}$. If $MAD_{k\%}$ differs too much and exceeds the threshold, errors can be taken to be non-constant and linearity may no longer hold

Illustration



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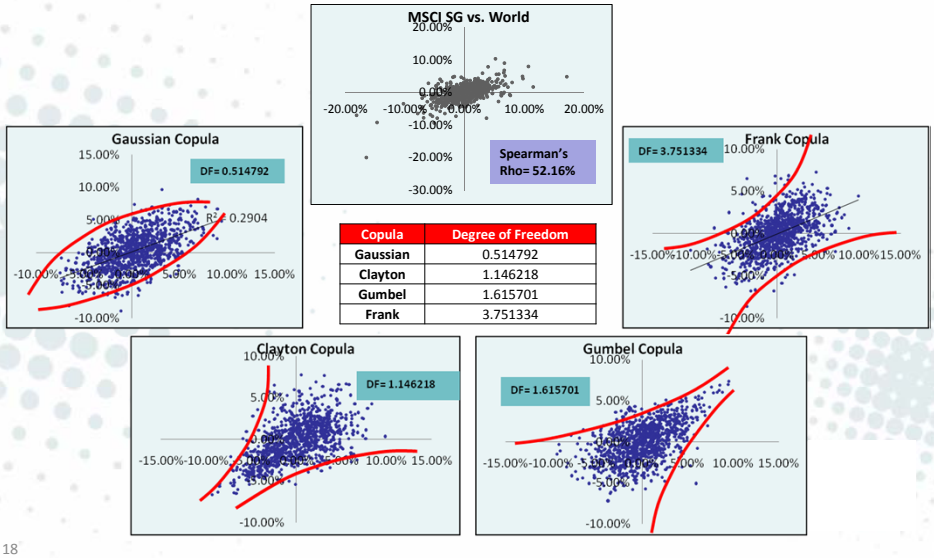
Section 1.2c: Curve Fitting MSCI SG vs. World

Objective: simulated Spearman's Rho equals to that of actual historical data



Copula	Degree of Freedom
Gaussian	0.514792
Clayton	1.146218
Gumbel	1.615701
Frank	3.751334

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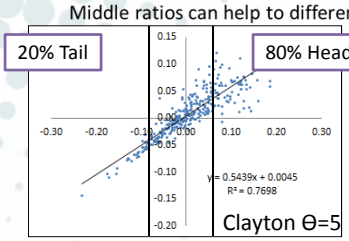
Section 1.2c: Curve Fitting Parameterized Filtering method

- Divide the data into 3 sections as before (using X-values): **LH Tail , Middle, RH Tail** (or Head)
  - Choose $k1$ and $k2$ percentiles to draw the boundaries e.g. 20% & 80%. Vary $k1$ and $k2$ to check robustness of results
  - Calculate the linear correlation ρ within each range, then calculate **Tail/Head**, **Tail/Middle** and **Head/Middle** correlation ratios.
- Copula identification by analyzing correlation ratios in tail and middle
  - To identify asymmetric copulas, criteria is that Tail/Head ρ should **differ significantly from 1**. E.g. Clayton, Tail correlation should be **much stronger** than Head ρ such that Tail/Head ratio >>1.
  - For symmetric copulas like Gaussian, Tail/Head should be close to 1. Tail/Middle & Head Middle ratios can help to differentiate the various symmetric copulas.



20% Tail ρ	= 90%
80% Head ρ	= 20%
Tail/Head ρ-Ratio	**= 4.5 >> 1**
Strong evidence for Clayton	

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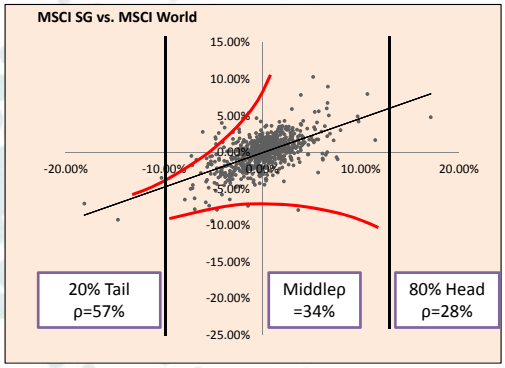
Section 1.2c: Back-Testing Result MSCI SG vs. World

Objective: simulated Spearman's Rho equal to that of actual data



Spearman's Rho

20% Tail ρ	= 57%
80% Head ρ	= 34%
20%-80% Middle ρ	= 28%
Tail/Head	**= 1.65**
Tail/Mid	**= 2.02**
Head/Mid	**= 0.72**

Limits	Gaussian	FGM	Frank	Gumbel	Clayton
Tail/Head Lower Limit	0.8	0.8	0.6	0.0	1.2
Tail/Head Upper Limit	1.2	1.2	1.4	0.8	100.0
Tail/Mid Lower Limit	0.6	0.6	0.0	0.0	1.0
Tail/Mid Upper Limit	1.1	1.1	0.7	1.0	100.0
Head/Mid Lower Limit	0.6	0.6	0.0	1.0	0.0
Head/Mid Upper Limit	1.1	1.1	0.7	100.0	1.0
20/80 Criteria	NO	NO	NO	NO	YES

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Section 1.2: Conclusion

Summary:

- *The relationship (e.g. within investment space) does not always present a linear form*
- *Using the MAD method will help to ascertain whether the relationship is linear or non-linear, but it requires a sufficient number of data points*
- *The parameterized filtering method is useful in selecting which copula can be best used for simulation, but it too needs a sufficient number of data points*

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Section 2a: Data & Methodology

Data

Data Set	No. of Years	Total Sample Size/ Data Points
Investments	1999-2013	753 data points
General Insurance	2005-2012	88 ~ 240 data points
Life Insurance	1985-2012	28

Note: The estimated correlation coefficients rely heavily on the number of data points available and hence may be quite volatile

Methodology

1.Calculate the empirical correlation coefficient (Rank Correlation) between risk factors

2.For Loss Ratio correlations, weight against exposure size to obtain industry average correlation

3.Calculate 95% confidence interval of coefficient

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Section 2b: Analysis – General Insurance

	Weakly Positive Correlation (10%-20%)	Uncorrelated
Macro Factors	GWP Growth(t) vs GDP Growth(t-1) GWP Growth(t) vs Inflation(t)	Loss Ratio vs Inflation Loss Ratio vs GDP Growth
Insurance Factors	Loss Ratio(t) vs GWP Growth(t-1)	
Investment Factors		Loss Ratios vs Equity Returns Loss Ratios vs Bond Yields Between Insurance Loss and Investment Loss
Loss Ratios	SIF Marine vs SIF Worker's Compensation SIF Fire vs SIF Personal Accident SIF Personal Accident vs SIF Worker's Compensation SIF Marine vs SIF Miscellaneous SIF Fire vs SIF Miscellaneous SIF Health vs SIF Miscellaneous	All Other lines in SIF All lines in OIF Between SIF Fund and OIF Fund

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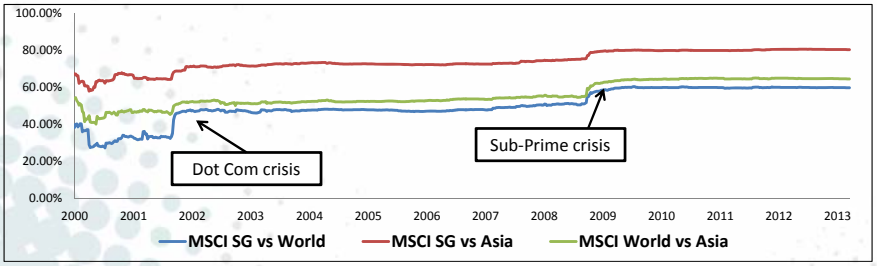
Section 2c: Analysis – Life and Investments Intra-Risk Public Equity

Historical Correlation of Singapore Equity Return vs. Rest of World & Asia Equities



Source: Morgan Stanley Capital International Index(MSCI)

Portfolio	Spearman Correlation	
	Pre & Post 2008 GFC	Pre 2008 GFC Only
MSCI Asia vs. MSCI World	60%	57%
MSCI Singapore vs. MSCI World	52%	48%
MSCI Singapore vs. MSCI Asia	76%	71%

Comments:

- *Equity correlation tends to spike after the crisis*

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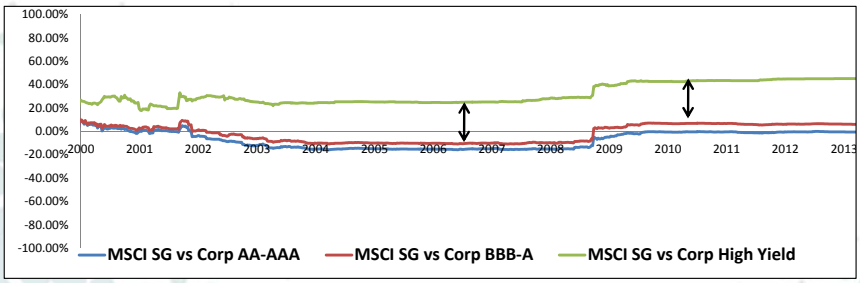
Section 2c: Analysis – Life and Investments Inter-Risk Bonds vs Public Equity

Historical Correlation of Singapore Equity Return vs. return of different rated corporate bonds



Source: Bank of America Merrill Lynch Index (BofA)

Portfolio	Spearman Correlation	
	Pre & Post 2008 GFC	Pre 2008 GFC Only
MSCI Singapore vs. BofA Corp High Yield	39%	27%
MSCI Singapore vs. BofA Corp BBB-A	-3%	-9%
MSCI Singapore vs. BofA Corp AA-AAA	-7%	-14%

Comments:

- *High yield bonds tend to have a high correlation with equity*
- *Highly rated bonds are less correlated with equity*

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Section 2c: Analysis – Life and Investments Inter-Risk – Corporate Bond and Equities

Spearman Correlation Table with 95% confidence interval (1999-2013)

		Corp Bonds									Equities					
		AA-AAA			BBB-A			High Yield			Singapore			Asia		
		Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp
Corp Bonds	AA-AAA															
	BBB-A	94%	**95%**	95%												
	High Yield	12%	**19%**	25%	22%	**29%**	36%									
Equities	Singapore	-14%	**-7%**	0%	-10%	**-3%**	4%	33%	**39%**	45%						
	Asia	-17%	**-9%**	-2%	-10%	**-3%**	4%	38%	**44%**	50%	73%	**76%**	79%			
	World	-26%	**-19%**	-13%	-22%	**-15%**	-8%	36%	**42%**	48%	47%	**52%**	57%	56%	**60%**	65%

Source: Equity Index: Morgan Stanley Capital International Index (MSCI)
Corp Bond Index: Bank of America Merrill Lynch Index

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Section 2c: Analysis – Life and Investments Inter-Risk - Government and Corporate Bond

Spearman Correlation Table with 95% confidence interval (1999-2013)

		Govt Bonds									Corp Bonds					
		UOB All			UOB Long			UOB Short			AA-AAA			BBB-A		
		Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp
Govt Bonds	UOB All															
	UOB Long	99%	**100%**	100%												
	UOB Short	64%	**68%**	72%	58%	**62%**	67%									
Corp Bonds	AA-AAA	31%	**37%**	43%	32%	**38%**	44%	14%	**21%**	28%						
	BBB-A	27%	**34%**	40%	28%	**34%**	41%	12%	**19%**	26%	94%	**95%**	95%			
	High Yield	-9%	**-2%**	5%	-9%	**-2%**	5%	-6%	**1%**	8%	12%	**19%**	25%	22%	**29%**	36%

Source: Govt Bond Index: UOB SGS Index . Corp Bond Index: Bank of America Merrill Lynch Index

Comments:

- *High yield bonds tend to be less correlated with Govt and Higher Rated bonds*

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Section 2c: Analysis – Life and Investments Inter-Risk – Govt Bond vs. Public Equity

Spearman Correlation Table with 95% confidence interval (1999-2013)

		Govt Bonds								
		UOB All			UOB Long			UOB Short		
		Low	Avg	Upp	Low	Avg	Upp	Low	Avg	Upp
Equities	Singapore	-20%	**-13%**	-5%	-20%	**-13%**	-6%	-13%	**-6%**	1%
	Asia	-22%	**-15%**	-8%	-22%	**-15%**	-8%	-14%	**-7%**	0%
	World	-29%	**-22%**	-15%	-29%	**-22%**	-16%	-19%	**-12%**	-5%

Source:
Govt Bond Index: UOB SGS Index
Public Equity Index: Morgan Stanley Capital International Index (MSCI)

Comments:

- Government Bond Indices tend to be negatively correlated with Equity Indices.

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Section 3.1: Impact Analysis – General Insurance

a. **Structure and Assumptions**

b. **Base Scenario: Mean Parameters with 10% Correlation Floor**

c. **Comparing Sensitivity Scenario Results**

1. Sensitivity Scenario 1: 95th Percentile Parameters
2. Sensitivity Scenario 2: 50% Inter-Risk Correlations (Insurance Risk and Investment Risk)
3. Sensitivity Scenario 3: Personal Line Insurer
4. Sensitivity Scenario 4: Mean Parameters with 0% Correlation Floor
5. Sensitivity Scenario 5: Mean Parameters with 25% Correlation Floor
6. Sensitivity Scenario 6: Mean Parameters with 50% Correlation Floor

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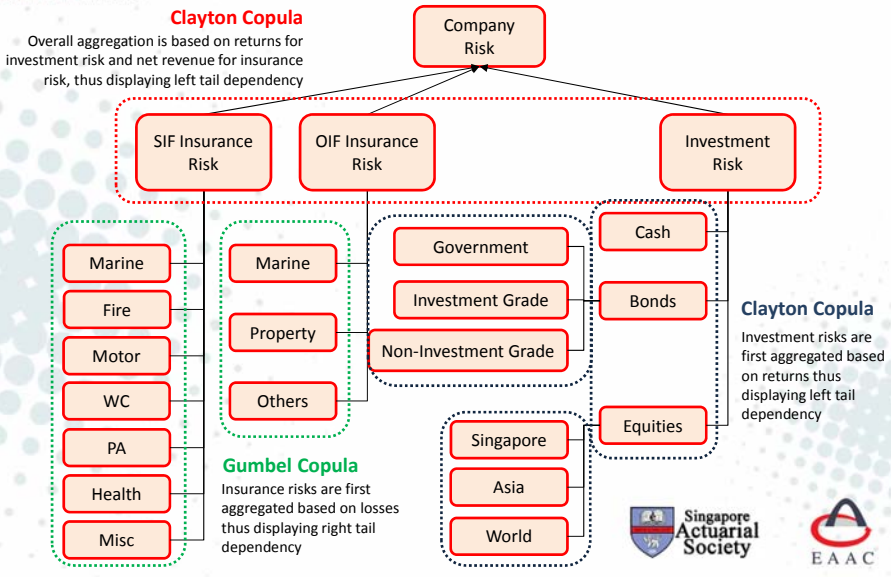
Section 3.1a: Structure – General Insurance



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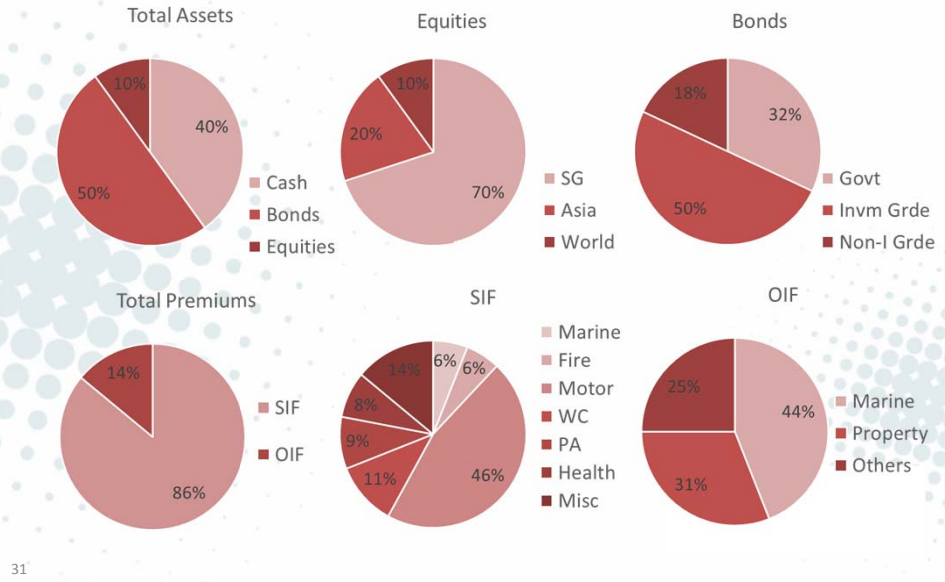
Section 3.1a: Assumptions – General Insurance



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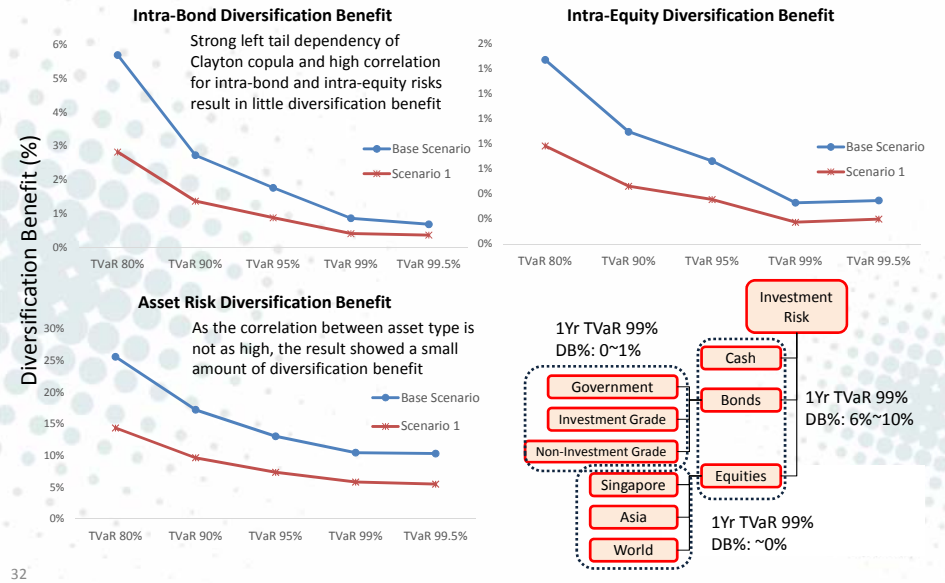
Section 3.1b: General Insurance Scenario 1 Intra-Risk Results – Investments



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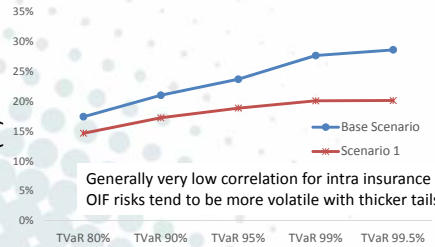
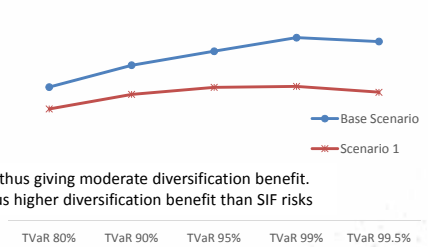
Section 3.1b: General Insurance Scenario 1 Intra-Risk Results – Insurance

SIF Intra-Risks Diversification Benefit

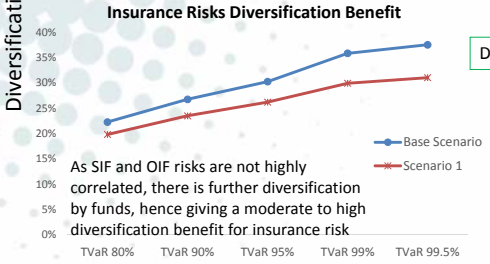


OIF Intra-Risks Diversification Benefit

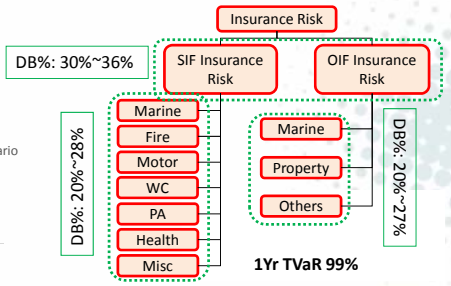


Generally very low correlation for intra insurance risk thus giving moderate diversification benefit.
OIF risks tend to be more volatile with thicker tails thus higher diversification benefit than SIF risks

Insurance Risks Diversification Benefit



As SIF and OIF risks are not highly correlated, there is further diversification by funds, hence giving a moderate to high diversification benefit for insurance risk



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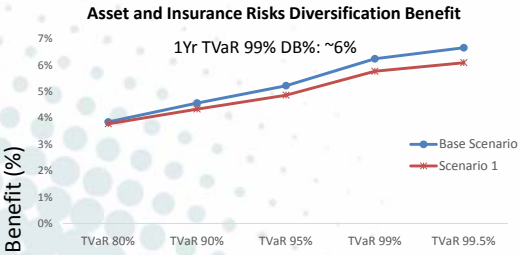
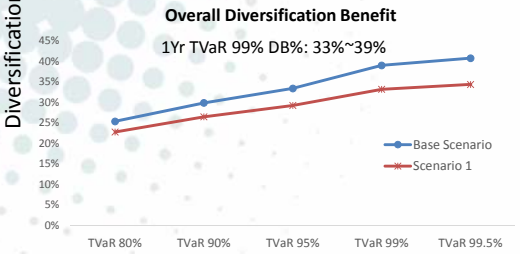
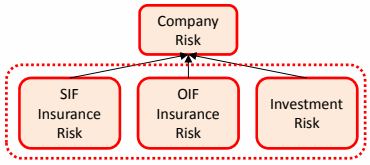
Section 3.1b: General Insurance Scenario 1 Inter-Risk Results

Asset and Insurance Risks Diversification Benefit

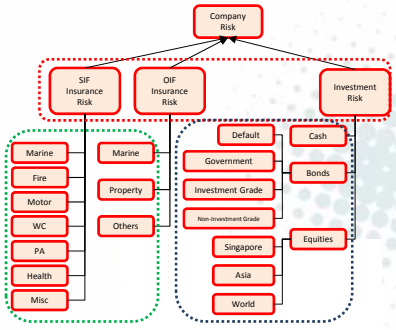


Overall Diversification Benefit





Weak inter-risk correlation gives more room for diversification and thus an overall moderate to high diversification benefit



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Section 3.1c: Comparison of Sensitivity Scenario Results

TVaR 99% Results	Base	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6
Diversification Benefit	10% Floor	95th Percentile	50% Inter Risk Correlation	Personal Line Insurer	0% Floor	25% Floor	50% Floor
Investment Risk	10%	6%	10%	6%	10%	10%	9%
Insurance Risk	36%	30%	30%	14%	38%	27%	12%
Overall	39%	33%	30%	22%	42%	29%	13%

Comments:

•Through scenario tests by increasing the floor correlation percentage, there is almost no effect on investment risk since it is already highly correlated above the floor. The greatest effect comes from wrong estimation of the intra insurance correlation and also inter risk correlation. However, even with a high floor of 50% correlation, we still estimated 13% diversification benefit.

•As expected, personal line insurer will have less diversification benefit as the risks are more concentrated in fewer lines, mainly in motor business.

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Section 3.2: Impact Analysis – Life & Investments

a. **Structure**
b. **Intra Equity Class Diversification Benefit**
c. **Intra Bond Diversification Benefit**
d. **Intra Product Line Diversification Benefit (life mortality)**
e. **Inter Asset Class Diversification Benefit**
f. **Portfolio Risk Diversification Benefit**

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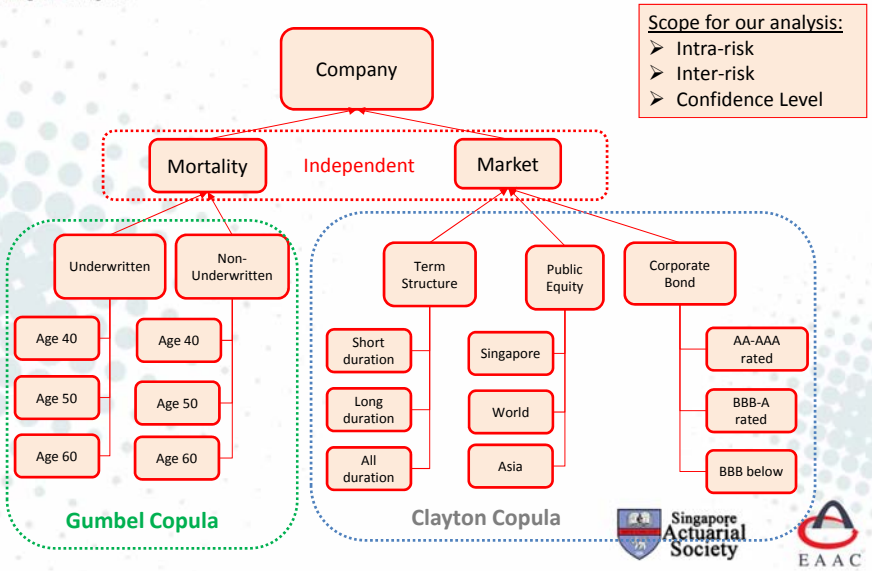
Section 3.2a: Structure – Life & Investments

Scope for our analysis:
- Intra-risk
- Inter-risk
- Confidence Level



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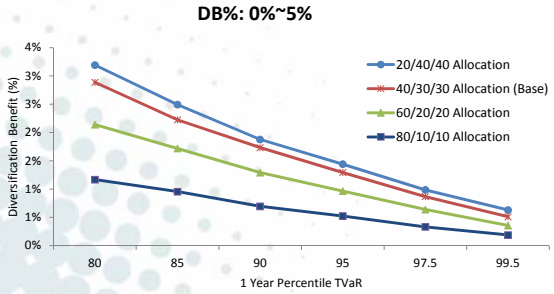
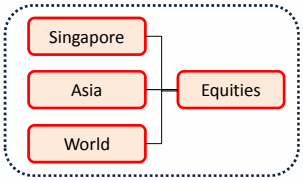
Section 3.2b: Intra Equity Class Diversification Benefit





Remarks: 40/30/30 refers to 40% in Sing, 30% in Asia and 30% in World

Comments:

- *Diversification benefit decreases when the equity market is distressed*
- *Reducing Singapore equity exposure to a well diversified portfolio such as Asia and World increases diversification benefit slightly*

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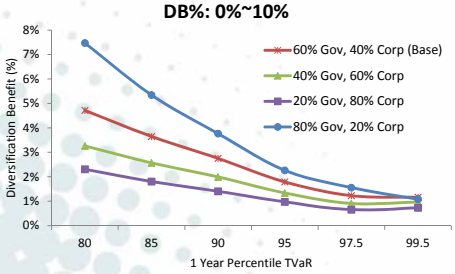
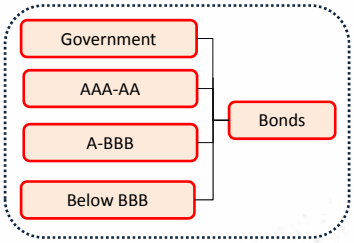
Section 3.2c: Intra Bond Diversification Benefit

DB%: 0%~10%





Remarks: 60/40 refers 60% in Government bonds ,40% in Corporate bond with equal weights in rating buckets

Comments:

- *Diversification benefit exists within bond asset class, but decreases when market is distressed*
- *Increasing government bond exposure increases diversification benefit*

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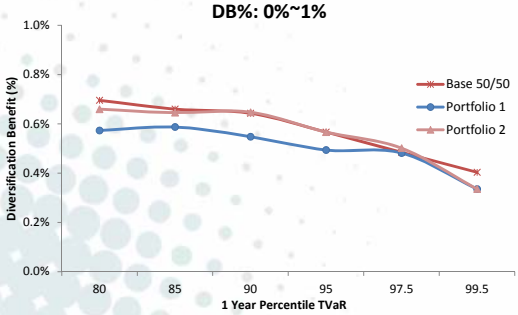
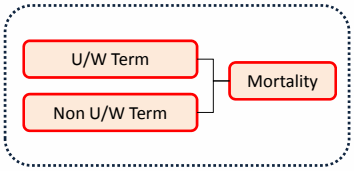
Section 3.2d: Intra Product Line Diversification Benefit (Life Mortality)

DB%: 0%~1%





Portfolio	*U/W Term*	*Non U/W Term*
Base	**50%**	**50%**
1	90%	10%
2	70%	30%

Comments:

- *A low diversification benefit observed within mortality business, it will decrease further when TVaR increases*
- *When an extreme event happens, e.g. at 99.5th percentile, changing mix has little impact to diversification benefit*

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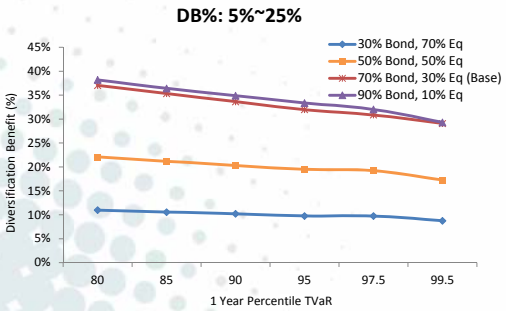
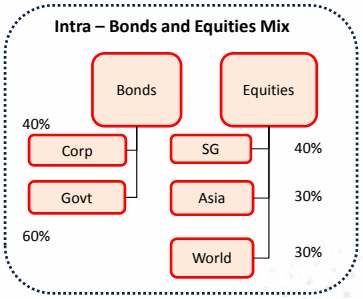
Section 3.2e: Inter Asset Class Diversification Benefit





Remark: Variation in bonds and equities mix, but the mix of intra-bond and – equities is assumed the same as above

Comments:

- *Diversification benefit exists between asset classes*
- *But it tends to be reduced when the market becomes distressed*
- *Increasing bond portfolio exposure improves diversification benefit*

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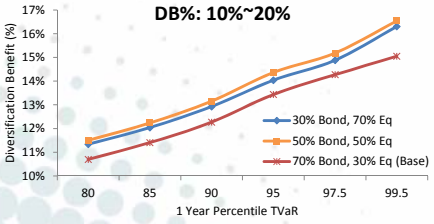
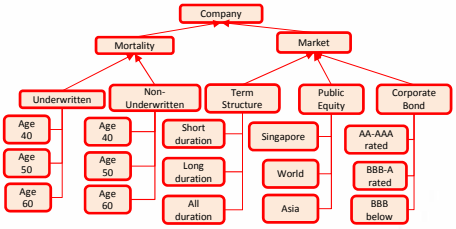
Section 3.2f: Portfolio Risk Diversification Benefit





Comments:

- *The increase in diversification benefit when TVaR increases is due to the fact that the mortality and market variables are assumed to be independent, which results in the large claim losses are unlikely to occur at the same time as large investment losses*

80th-99.5th percentile Diversification Benefit Table

	Equity		Credit		Mortality		Portfolio	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Equity	0%	5%						
Credit			0%	5%				
Mortality					0%	5%		
Portfolio							5%	20%

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